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ON A MODELLING OF THE GASLIFT PROCESS BY THE SYSTEM OF DELAY ARGUMENT DIFFERENTIAL EQUATIONS

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Abstract. In the work the problem of control of the gaslift process is considered. In differ from existing works here is assumed that the flow rate of the pump-compressor tubes at the bottom of the well is determined by the flow in the annular space around of the pipe with a certain delay. The considered problem is reduced to the delay argument optimal control problem.

Keywords: Gaslift, delay argument, optimal control, Euler-Lagrange equations.

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1 Introduction

It is known that when oil well is put into operation, oil is pumped to the surface by fountain method. But as time goes on, the fountain stops as the pressure of the well drops. In this case, it is necessary to switch to mechanical methods to extract oil from the layer. One of such methods is the exploitation of oil wells by the gaslift method. The essence of this method is that by injecting additional gas into the well, the bottom of the well produces a liquid-gas mixture with more light specific weight, which allows the oil to flow out of the well.

Different works have been devoted to the modeling of the gaslift method (Barashkin & Samarin, 2006; Sakharov & Mokhov, 2004; Sakharov et al., 1985; Charny, 1951). One of the models that adequately describes this process is the mathematical model proposed in Aliev et al. (2010a). In Aliev & Mutallimov (2009), the optimal control problem for the gaslift method is set based on this model. In this model, the flow and pressure at the bottom of the well are assumed to be constant.

This problem is reduced to the linear-quadratic optimal control problem and its solution method is developed. As a result, the program trajectory and control was found. In Aliev et al. (2010b), the problem of construction of the optimal stabilizer around the program trajectory and controller i.e. optimal stabilization problem is solved. This allows one to keep the desired debit stable for some period. Here in differ from Barashkin & Samarin (2006) we assume that the flow rate of the pump-compressor tubes at the bottom of the well is determined by the flow in the annular space around of the pipe with a certain delay and the problem is reduced to the delay argument optimal control problem.

2 Formulation of the problem

It is known that (Aliev et al., 2010a), the mathematical model of the operation of the gaslift well is approximately described by the by the following system of the linear partial differential equations

$$\begin{cases} \frac{\partial P}{\partial t} = -\frac{c^2}{F}\frac{\partial Q}{\partial x} \\ \\ \frac{\partial Q}{\partial t} = -\bar{F}\frac{\partial P}{\partial x} - 2aQ. \end{cases} \tag{1}$$

here $t \geq 0, \;\; x \in [0, 2L]$.

In Barashkin & Samarin (2006) system (1) is reduced to the system of ordinary differential equations by dividing L by N equal parts $l = L/N$. For the case $N = 2$ in the annular domain we get the following system of the ordinary differential equations

$$\begin{cases} \dot{P}_1(t) = -\frac{c_1^2}{F_1 l}Q_1(t) + \frac{c_1^2}{F_1 l}Q_0(t), \\ \\ \dot{Q}_1(t) = -\frac{F_1}{l}P_1(t) + \frac{F_1}{l}P_0(t) - 2a_1Q_1(t), \\ \\ \dot{P}_2(t) = -\frac{c_1^2}{F_1 l}Q_2(t) + \frac{c_1^2}{F_1 l}Q_1(t), \\ \\ \dot{Q}_2(t) = -\frac{F_1}{l}P_2(t) + \frac{F_1}{l}P_1(t) - 2a_1Q_2(t). \end{cases} \tag{2}$$

Here the functions $Q_0\,(t)\,,\;P_0\,(t)$ are the volume of the injected gas to the annular domain and its use correspondingly, and gaslift process is controlled by these functions. As is noted in Barashkin & Samarin (2006) $Q_2\,(t)$ and$P_2\,(t)$ are the values of these functions correspondingly, in the bottom of the well. It is clear that the pressure and use formed that in the bottom of the lifting pipe of the well depends on their values in the annular at the argument t with the delay τ i.e.

$$\begin{aligned} \bar{Q}_2(t) &= \alpha Q_2(t-\tau), \\ \bar{P}_2(t) &= \beta P_2(t-\tau), \end{aligned} \tag{3}$$

Considering this for the lifting pipe we get the following equations

$$\begin{cases} \dot{P}_3(t) = -\frac{c_2^2}{F_2 l}Q_3(t) + \frac{c_2^2}{F_2 l}\alpha Q_2(t-\tau), \\ \\ \dot{Q}_3(t) = -\frac{F_2}{l}P_3(t) + \frac{F_2}{l}\beta P_2(t-\tau) - 2a_2Q_3(t), \\ \\ \dot{P}_4(t) = -\frac{c_2^2}{F_2 l}Q_4(t) + \frac{c_2^2}{F_2 l}Q_3(t), \\ \\ \dot{Q}_4(t) = -\frac{F_2}{l}P_4(t) + \frac{F_2}{l}P_3(t) - 2a_2Q_4(t). \end{cases} \tag{4}$$

Thus we have the system of the first order differential equations (2), (4). The corresponding initial conditions have the form

$$P_k\,(t_0) = P_k^0, \quad Q_k\,(t_0) = Q_k^0 \qquad k = \overline{1,4} \tag{5}$$

Let us denote

$$x(t) = [P_1(t), Q_1(t), P_2(t), Q_2(t), P_3(t), Q_3(t), P_4(t), Q_4(t)]',$$

$$x^0 = \left[P_1^0, Q_1^0, P_2^0, Q_2^0, P_3^0, Q_3^0, P_4^0, Q_4^0\right]',$$

$$u(t) = [Q_0(t), P_0(t)]'$$

and

$$A=\begin{bmatrix} 0 & \frac{c_1^2}{F_1 l} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{F_1}{l} & -2a_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{c_1^2}{F_1 l} & 0 & -\frac{c_1^2}{F_1 l} & 0 & 0 & 0 & 0 \\ \frac{F_1}{l} & 0 & -\frac{F_1}{l} & -2a_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{c_2^2}{F_2 l} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{F_2}{l} & -2a_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{c_2^2}{F_2 l} & 0 & -\frac{c_2^2}{F_2 l} \\ 0 & 0 & 0 & 0 & \frac{F_2}{l} & 0 & -\frac{F_2}{l} & -2a_2 \end{bmatrix},$$

$$B=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha\frac{c_2^2}{F_2 l} & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta\frac{F_2}{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad G=\begin{bmatrix} \frac{c_1^2}{F_1 l} & 0 \\ 0 & \frac{F_1}{l} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Then from expressions (2), (4) and (5) we have

$$\dot{x}(t) = Ax(t) + Bx(t-\tau) + Gu(t), \tag{6}$$

$$x(0) = x^0. \tag{7}$$

Here $u(t)$ is considered as a control parameter.

The problem is: to minimize the functional

$$J = \frac{1}{2}\left[Q_4(T) - \bar{Q}\right]^2 + \frac{1}{2}\int_0^T \left\{x'(t)Rx(t) + u'(t)Cu(t)\right\} dt \tag{8}$$

Subject to (6), (7).

This is a linear-quadratic optimal control problem with $R' = R > 0,\ C' = C > 0$.

As we see problem (6)-(8) is an optimal control problem with delay argument. First we should identify the parameter τ and coefficients $\alpha,\ \beta$. For this purpose suppose that in the gaslift process the values $Q_2(t)$ and $P_2(t)$ of the functions $Q_0(t)$ and $P_0(t)$ at the point $t = t_i$ are known. We find the parameter τ using these values. Thus the problem is reduced to the identification problems that may be solved by some known method.

As is noted in Barashkin & Samarin (2006) denoting

$$N=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha \end{bmatrix} \bar{x}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \bar{Q} \end{bmatrix}, \tag{9}$$

functional (8) may be written in the form

$$.J = \frac{1}{2}\left(x(T) - \bar{x}\right)' N \left(x(T) - \bar{x}\right) + \frac{1}{2}\int_0^T \left\{x'(t)Rx(t) + u'(t)Cu(t)\right\} dt \tag{10}$$

3 Euler-Lagrange equations

Now we solve optimal control problem (6), (7), (10). First we construct the extended functional in the form

$$\begin{array}{l}\bar{\mathrm{J}} = \frac{1}{2}\left[x(T)-\bar{x}\right]' N \left[x(T)-\bar{x}\right] + \int\limits_0^T \left\{ \frac{1}{2}\left[x'(t)Rx(t) + u'(t)Cu(t)\right] + \right. \\ \left. + \lambda'(t)\left[A(x(t) + B(t-\tau) + Gu(t) - \dot{x}(t)\right]\right\}. \end{array} \tag{11}$$

Using the transformation

$$-\int\limits_0^T \lambda'(t)\,\dot{x}(t)dt = -\int\limits_0^T \lambda'(t)dx(t) = -\lambda'(t)x(t)\Big|_0^T + \int\limits_0^T \dot{\lambda}(t)x(t)dt = -\lambda'(T)x(T) + \lambda'(0)x(0)+ \tag{12}$$

We can write functional (11) as follows

$$\begin{array}{l}\hat{J} = \frac{1}{2}\left[x(T)-\hat{x}\right] N \left[x(T)-\hat{x}\right] - \lambda\prime(T)x(t) + \lambda\prime(0)x(0) + \\ + \int_0^T \left\{ \frac{1}{2}\left[x\prime(t)Rx(t) + u\prime(t)Cu(t)\right] + + \lambda\prime(t)\left[Ax(t) + Bx(t-\tau) + Gu(t) + \lambda\prime(t)x(t)\right] \right\} dt. \end{array} \tag{13}$$

If to use the equality

$$\int_0^T \lambda'(t)Bx(t-\tau)dt = \int_{-\tau}^{T-\tau} \lambda'(t+\tau)Bx(t)dt \tag{14}$$

then the functional $\bar{J}$ takes the form

$$\begin{array}{l}\hat{J} = \frac{1}{2}\left[x(T)-\hat{x}\right] N \left[x(T)-\hat{x}\right] - \lambda\prime(T)x(t) + \lambda\prime(0)x(0) + \\ + \int_0^T \left\{ \frac{1}{2}\left[x\prime(t)Rx(t) + u\prime(t)Cu(t)\right] + + \lambda\prime(t)\left[Ax(t) + Gu(t)\right] + \dot{\lambda}\prime(t)x(t) \right\} dt + \\ + \int_{-\tau}^{T-\tau} \lambda\prime(t+\tau)Bx(t)dt. \end{array} \tag{15}$$

Equaling the first variation of this functional to zero from the equalities

$$\frac{\partial \bar{J}}{\partial x(t)} = 0\ , \quad \frac{\partial \bar{J}}{\partial u(t)} = 0\,, \quad \frac{\partial \bar{J}}{\partial x(T)} = 0$$

we get the following expressions

$$\begin{array}{l} x'(t)R + \lambda' A + \dot{\lambda}'(t) + \lambda'(t+\tau)B = 0, \\ u'(t)C + \lambda'(t)G = 0, \\ \left[x(T)-\bar{x}\right]' N - \lambda'(T) = 0. \end{array}$$

From the last we get

$$\lambda'(t) = -Rx(t) - A'\lambda(t) - B'\lambda(t+\tau) \tag{16}$$

$$\lambda(T) = N'\left[x(T)-\bar{x}\right] \tag{17}$$

$$u(t) = -c^{-1}G'\lambda(t) \tag{18}$$

Equation (16), (17) together with relations (6), (7) lead us to the following Euler-Lagrange equation

$$x'(t) = Ax(t) + Bx(t-\tau) - GC^{-1}G'\lambda(t), \tag{19}$$

$$\lambda'(t) = -Rx(t) - A'\lambda(t) - B'\lambda(t+\tau), \tag{20}$$

$$x(0) = x^0, \tag{21}$$

$$\lambda(T) = N'\left[x(T) - \bar{x}\right]. \tag{22}$$

4 Conclusion

Apparently, the considered optimal control problem for the gaslift process is reduced to the linear-quadratic optimal control with a delayed argument, which can be solved and the program trajectory and controller can be found.

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